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LETTER TO THE EDITOR

Propagation of three-dimensional solitary pulses in waveguide channels with cylindrical symmetry

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Abstract. The solitary-wave solution of the (3+1)-dimensional non-linear Schrödinger equation is obtained, taking into account the energy exchange between the central and peripheral parts of the pulse inside and outside the induced waveguide in a non-linear medium.

The real setting of the self-channelling problem is the description of the three-dimensional pulse evolution in non-linear media. The main difficulty in this is connected with the conclusion derived in a series of works (Zakharov and Kuznetsov 1986), on the instability of three-dimensional solitons and the possibility of collapse in the plasma, as well as in optical fibres. However, as was demonstrated in Lomov and Rabinovich (1988), three-dimensional solitary waves may exist in open systems, i.e. along with the interaction of non-linear systems with external dissipative effects. Another case of an open system connected with inter-mode interaction is considered in Afanasiev *et al* (1988). Hence the self-channelling description requires taking into account not only the active losses, usually known to be small, but also the 'dynamic losses', connected with energy exchange via side-boundaries of the waveguide channel, rising dynamically in the pulse self-organisation process. In this letter we consider one more possibility of an open system appearing naturally in a single-mode regime that could be named the soliton 'self-supply'. As the transverse structure of a laser beam is non-uniform and is usually approximated by a Gaussian field density distribution, in non-linear propagation only the central part of the beam takes part, whereas the propagation of the pulse periphery might be described by the linear parabolic equation for slowly changing amplitude. Taking the power distribution in the channel cross section in the form:

$$P(\tau) = P(0)\exp(-\tau^2/\tau_0^2)$$

($P(0) = |E(0)|^2/4\pi\Delta t$ is the peak pulse power, Δt being the pulse duration) and applying the concept of critical self-channelling power $P_{c\tau}$, we obtain the estimate for the 'effective channel radius':

$$\tau_1^2 = \tau_0^2 \ln[P(0)/P_{c\tau}].$$

As in the self-channelling process a deformation of the 'boundaries' between the linear and non-linear areas takes place, there is a partial transfer of energy from the external area into the channel, which can be characterised by the coupling index a. The energy transferred from the canalised area into the external area (characterised by index b)

will be much less, in agreement with the self-channelling conditions. Thus $a|\phi_{\tau=\tau_1}|^2$ is the energy transferred into the channel (where ϕ is the field outside the channel area), whilst $b|\psi_{\tau=\tau_1}|^2$ (where ψ is the field inside the channel) accounts for the dynamic losses, which could be neglected (in Afanasiev *et al* (1988) they are connected with intensive modes of the field inside the channel). The field ϕ obeys the linear equation

$$2\mathrm{i}\frac{\partial\phi}{\partial z}+\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial\tau^2}+\frac{1}{\tau}\frac{\partial\phi}{\partial\tau}+\frac{1}{\tau^2}\frac{\partial^2\phi}{\partial\varphi^2}=0 \tag{1}$$

the solution of which is of the form (Marcuse 1972) (the choice of this solution corresponds to the condition $R(\tau,\varphi)\to 0$ as $\tau\to\infty$ and the fact that at $\tau\sim 0$ there exists the focus; hence the plane-wave approximation cannot be used near the boundary):

$$\phi(z,x,\tau,\varphi)=\frac{\phi_0}{\sqrt{z+z_0}}\exp\left[\frac{\mathrm{i}}{2}\left(\frac{x^2}{z+z_0}-\kappa^2 z\right)\right]R(\tau,\varphi)$$

where z_0 is the 'focus' of the induced lens (Chiao *et al* 1964), henceforth to be used as the channel scale zero point; $R(\tau,\varphi)$ is the radial equation solution in cylindrical variables; κ is the parameter of variable decoupling; $x=t-z/v$ (v is the pulse group velocity). The field amplitude might be normed by the inverse coupling parameter (i.e. the renormed amplitude represents the part of external field which spreads into the channel).

The self-channelling process considered from the open system viewpoint is described by the non-linear Schrödinger equation that has, in non-dimensional variables and assuming cyclindrical symmetry and assuming the presence of the amplifying term (Hasegawa and Kodama 1981), the following form:

$$\mathrm{i}\frac{\partial\psi}{\partial\xi}+\frac{\delta}{2}\frac{\partial^2\psi}{\partial\tau^2}+\frac{1}{2}\frac{\partial^2\psi}{\partial\rho^2}+\frac{1}{2\rho}\frac{\partial\psi}{\partial\rho}+|\psi|^2\psi-\frac{\mathrm{i}}{\xi_0+\xi}\psi=0. \tag{2}$$

For the case of pulse propagation in optical fibres these variables have the form (Abdulaev *et al* 1987)

$$\psi=\frac{E}{E(0)}\qquad \tau=\left(\frac{kh_2|E(0)|^2}{h_0|k''|}\right)^{1/2}(t-z/v)\qquad \delta=\operatorname{sgn}(-k'')$$

$$\xi=\frac{h_2k|E(0)|^2}{h_0}z\qquad \rho=\left(\frac{kh_2|E(0)|^2}{h_0}\right)^{1/2}\tau$$

whereas the coupling index a is normed by the following condition:

$$\frac{a}{2}\int_0^{2\pi}|\phi_0R(\tau,\varphi)|^2_{\tau=\tau_1}\,\mathrm{d}\varphi=1.$$

The direct insertion demonstrates that equation (2) has the following exact solution in the form of the stable three-dimensional (in coordinates) soliton ($\psi_0=\psi(0)$):

$$\psi(\xi,\tau,\rho)=\psi_0\operatorname{sech}(\psi_0\tau)\exp\left[\frac{\mathrm{i}}{2}\left(\psi_0^2\xi+\frac{\rho^2}{\xi_0+\xi}\right)\right] \tag{3}$$

for $\delta=1$, and for $\delta=-1$ we have the kink-form solution

$$\psi(\xi,\tau,\rho)=\psi_0\tanh(\psi_0\tau)\exp\left[\mathrm{i}\left(\psi_0^2\xi+\frac{\rho^2}{2(\xi_0+\xi)}\right)\right]. \tag{4}$$

These solutions determine the pulse form after the self-channelling appearance and they have the form of a solitary wave pulsing in the radial direction, with this pulsation gradually disappearing when $\xi \to \infty$. This field dependence on ρ is of the general character and depends upon the choice of the field profile (Zakharov and Synach 1975). As can be seen from (3) and (4), these solutions are being factorised, so that the radial part is a limited function with module equal to 1, whereas the longitudinal part is the solution of the one-dimensional NLS equation—its stability has been shown in many papers (Abdulaev *et al* 1987). As can be seen from (3) and (4), the field intensity in the established channel no longer depends upon the transverse field structure.

If we consider (2) with the losses freely dependent on time (i.e. with the change of the last term by $\mathrm{i}\Gamma(\xi)\psi$) it is possible to see that its exact solution in the form of the modulated packet is

$$\psi(\xi, \tau, \rho) = \psi_0(\xi_0 + \xi)^{-1} \exp\left(-\int_0^\xi \Gamma(\xi')\, \mathrm{d}\xi' + \mathrm{i}\eta \right) \qquad (5)$$

where

$$\eta(\xi, \tau, \rho) = \eta_0 - \frac{\delta\lambda^2}{2}\xi + \lambda\tau + \frac{\rho^2}{2(\xi_0+\xi)} + \psi_0^2 \int \frac{\exp(-2\int_0^{\xi'} \Gamma(\xi'')\, \mathrm{d}\xi'')}{(\xi_0+\xi')^2}\, \mathrm{d}\xi'$$

(λ is the parameter of variable decoupling) will be non-blurring only in the case of the following choice:

$$\Gamma(\xi) = -\frac{1}{\xi_0 + \xi}. \qquad (6)$$

It is worth mentioning that such a form of $\Gamma(\xi)$, as was found for the PKdV equation in Baby (1987), is necessary for the Painlevé property to exist. It is to be expected that the property of non-blurring of the wave packets is essentially coupled with the dissipation function being in the form of (6). From the consideration of the problem carried out above, it is found that, in the case of the particular profile of the transverse structure of the input pulse, the field structure outside the waveguide channel essentially influences the propagation distance and the properties of the optical soliton.

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